Session 1, Paper 1

How to take into account potential change of the deterioration mode in Condition-Based Maintenance decision rule

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Abstract

Classical results in maintenance optimization are based on the characterization of an average system degradation behaviour. The main reasons of such approaches are the robustness of the decisions with respect to the system state and the time horizon. By cons, the "smoothing" effect in existing degradation models and the ``generic" aspect of the decision affect the decision quality as these models do not take into account potential changes in deterioration modes. Such consideration can be particularly valuable in a safety context. We propose here a condition-based maintenance approach which adapts the maintenance decision according to the degradation behaviour updated using current state observations. First, we propose to extend the system state definition, usually limited to an indicator of observable degradation by adding information related to the speed of deterioration referred hereafter as potential of degradation or deterioration growth rate. Moreover, we propose to take benefit of current observations as well as performed actions to update the degradation laws. More than the model fitness improvement, another advantage of our approach is the proposition of a more realistic modelling of the maintenance impact onto the future behaviour of the system. However, the introduction of the potential of degradation as a new decision parameter does not prejudge an intuitive structure for the decision policy. Moreover, as the classical concept of the failure rate, the potential of degradation is non observable. which implies more efforts in both optimization modelling and solution procedure. We highlight the structural properties of the optimization problem which ensure an optimal control limit policy. We will conclude our communication by the illustration of the results of our maintenance model in a pavement management context.

Keywords bivariate deterioration process, gamma process, imperfect maintenance, Markov decision process, partially observed Markov decision process.

Introduction

During the last decades, many companies, especially those that have invested few years ago in very expensive structures such energy production or civil engineering structures, have shown an increasing interest and need to define optimal maintenance strategies. Consequently, many researches have been developed and a large number of published papers can be found for maintenance optimization approaches [10].

Defining optimal maintenance strategies is usually a complex task. From one hand, maintenance activities usually tend to slow down company production and revenues and many constraints can then be imposed to meet meet required demands and revenues. From the other hand, the deterioration of production or service structures is a stochastic process that depends on many external agents such as loads, stresses, environmental conditions, etc, which may lead many approaches to be expensive and ``overprotective'' in order to manage the random risk of excessive deterioration levels such as block-replacement policies [4,6]. Hence, for an optimal policy to be efficient, a good knowledge upon the system deterioration process should be build and an accurate stochastic deterioration model should be developed for the optimization of maintenance policies.

We consider the maintenance optimization problem of cumulatively and stochastically deteriorating systems. The deterioration process of such system is usually a two-phase process which means that an underlying deterioration that is causing the observable deterioration should be taken into account. In practice, evaluation of deterioration levels of these systems is usually based on an observable indicator which is insufficient to reflect the true current deterioration level (especially in the first deterioration phase where no damage can be observed), as well as to predict future deterioration evolution.

In this work, we propose to introduce an novel deterioration indicator for the non-observable deterioration in both deterioration phases called the deterioration growth rate. Based on both observable and non-observable indicators, we develop a bivariate, stochastic deterioration process that highlight the dependence between observable and underlying deterioration. A special future about our deterioration model is being state-dependent in that its evolution law depends on the current deterioration level given by both indicators. We incorporate this deterioration model into a Markov decision process (MDP) to derive optimal maintenance policies. However, the MDP formulation implies that the system state, i.e., deterioration level can be fully observed, which is not the case of the deterioration growth rate. We extend the MDP formulation to a partially observed Markov decision process (POMDP) in order to take into account the non-deterministic knowledge that decision maker may have upon the deterioration growth rate.

Problem statement

Consider a system that is subject to continuous and stochastic deterioration because of variable loads and random stresses, in addition to aging. Such deterioration is usually a cumulative and two-phase process. More specifically, the first phase is the initiation phase during which, although no deterioration can be observed, the system is not as good as new. The second phase is the propagation phase that begins with the arrival of the first observable deterioration. this phase is characterized by both observable and underlying deterioration.

The system is periodically inspected at the beginning of equal decision epochs of length τ to yield measures of its observable deterioration level. Typically, this deterioration metric is a percentage of deterioration such as in the case of cracking or corroding structures, representing the ratio of deteriorated length or surface over the total one. Let ρ denote the observable deterioration parameter. Based on inspection outcomes, the decision-maker choose whether to do nothing (DN) or to perform one of several maintenance actions including perfect and imperfect actions. Maintaining perfectly or imperfectly the system conceals observable deterioration. Nevertheless, only the perfect action resets the system to an as-good-as-new (AGAN) state, whereas imperfect actions reset it to a state between the state just before maintenance and the AGAN one.

Let R be the number of available maintenance actions with $k=1,\ldots,R-1$ are imperfect actions and $k=R$ is the perfect maintenance. Maintaining the system incurs a maintenance cost $c(k)$ verifying $c(1)<..<c(R-1)\ll c(R)$. Moreover, in order to prevent the system from reaching high levels of deterioration and avoid selecting the *DN* action in order to minimize maintenance costs, a penalty cost called quality cost and denoted $C_q(\tau;.)$ is considered. We assume that the quality cost is a cumulated cost that is incurred at the beginning of each decision epoch and is function of the system deterioration level at its beginning. The objective is to define optimal maintenance policies to minimize the total discounted maintenance cost-to-go over the infinite horizon. Recall that a maintenance policy is a mapping from the state set to the action set that associates to each possibly observed state the optimal action to perform.

However, in a decision making problem under uncertainty, the quality of the optimal solution depends on the stochastic model used for uncertainty prediction. Thus, before formulating the maintenance optimization problem, we propose to develop a stochastic model for the deterioration evolution.

The main two limits that raise from ρ being the unique deterioration metric are the following. (i) The deterioration percentage does not take into account the underlying deterioration in both deterioration phases. Moreover, (ii) it cannot model different impacts of imperfect maintenances since they all reset it to zero.

We propose to introduce a second deterioration parameter called the deterioration growth rate (DGR) and denoted θ. This parameter represents a potential of (observable) deterioration in the initiation phase and an instantaneous deterioration speed in the propagation phase. In addition to taking into account underlying deterioration, the introduction of the DGR θ allows modeling different effects of imperfect maintenances on the system (unlike ρ that is reset to zero after any maintenance). More specifically, we assume that maintaining the system reduces its DGR to a level between its DGR just before maintenance and θ_0 that is the DGR of a new system. Let $\phi(s,k)$ be the deterministic maintenance effect function on the DGR when action k is performed in state $s=(\rho,\theta)$.

We develop a bivariate and state-dependent deterioration process to model joint evolution of observable and non-observable deterioration of the bivariate stochastic process $\{(\rho_t, \theta_t), t \geq 0\}$. The model is called state-dependent because its evolution law is dependent on the current deterioration level. Therefore, the stationarity assumption is relaxed with respect to state and not age. Moreover, we suppose imperfect maintenance action do not only affect the deterioration level but also the system behavior. More specifically, we relax the assumption of the same deterioration process identical to the new system one by considering a new deterioration process. This is made possible by making the deterioration law depending on the last maintenance type in addition to the resulting state from maintenance, denoted m. In the following, we present briefly the proposed deterioration model. For details, the reader can refer to [13].

The deterioration model

A. The initiation phase

Although no deterioration can be observed during the initiation phase, underlying deterioration represented by θ keeps on increasing until causing the arrival of the first observable deterioration. We consider the process of the first observable deterioration arrival which we model as a θ-dependent Poisson-process, i.e., with a rate function that depends on the DGR θ. Whereas, the underlying deterioration θ is a non-decreasing process that we model using a stationary gamma process.

B. The propagation phase

Cumulative deterioration processes is usually modeled using a gamma process ([1,8,7] and [9]) that models jumping and non-decreasing phenomena. However, most of the models assume that the process is stationary, i.e. that the increments modeling the process evolution do not depend on the age system. This assumption is restrictive especially for cumulative deteriorating systems for which deterioration is accelerated as the system ages. In order to relax this assumption, gamma process was extended to the generalized gamma process [5] that has an age-dependent evolution law. In a maintenance optimization context, the generalization of the gamma process incurs problem inextricability [5].

In this work, we propose to relax the assumption of stationary increments in state instead of age. More specifically, we make the evolution law of the gamma process during each time period depend on the system state at its beginning given by (ρ, θ). We term the resulting process as the ``stat-dependent'' gamma (SDG) process. Note that the evolution of the observable deterioration process does not only depend on its current level but also on the instantaneous speed of deterioration θ. This dependance on the deterioration growth rate models the effect of underlying deterioration on the observable one, allows differentiating individual deterioration behavior from a reference one. Since increments of the DGR can be positive as well as negative, we propose to model θ by a bilateral gamma (BG) process.

In an imperfect maintenance context, most of existing models restrict imperfect maintenance effects to the reduction of the system deterioration level. We relax this restrictive assumption by changing the deterioration process after each maintenance. This is made possible by making the deterioration evolution law depend also on the type of the last performed maintenance m. Finally, the dependance between evolution of the two deterioration parameters ρ and θ is given by the following joint density distribution:

$$h(x,y;\rho,\theta,m)=g(x;\rho,\theta,m)\,f(y;\rho,x,m) \qquad (1)$$

where $h(x,y;\rho,\theta,m)$ is the joint probability density function of the bivariate process $\{(\rho_t,\theta_t),t\geq 0\}$ conditionally to the current system state (ρ,θ,m), whereas $g(x;\rho,\theta,m)$ and $f(y;\rho,x,m)$ are conditional evolution laws of processes $\{\rho_t,t\geq 0\}$ and $\{\theta_t,t\geq 0\}$, respectively. From Equation (1), increments of the observable deterioration depend on both current levels of ρ and θ whereas the DGR increments depend on the observable deterioration level ρ and its increments x.

Optimizing maintenance decisions: fully and partially observed problems

The problem of optimizing maintenance decisions for stochastically deteriorating systems at the beginning of each decision epoch in order to minimize total expected maintenance cost over the infinite horizon can be formulated as a Markov decision process. However, the MDP formulation assumes that the state is fully observable, which is not the case of the DGR θ. We first approximate the DGR by the average speed of observable deterioration using successive observations of ρ and formulate the decision problem as an MDP that we solve using the policy iteration algorithm (PIA) [3].

Let $p^k_{ss'}$ denote the transition probability from state s to s' given that action k was performed. For a given last performed maintenance type $m=1,\ldots,R$, let $V(s)$ denote the optimal cost-to-go over the infinite horizon in state $s=(\rho,\theta)$ given by the following:

$$V(s)=c_0+min\{DN(s),min_{k=1,\ldots,R}MX_k(s)\} \qquad (2)$$

where,

$$DN(s)=C_q(\tau;s)+\lambda\sum_{s'}p^m_{ss'}V(s') \qquad (3)$$

$$MX_k(s)=c(k)+C_q(\tau;s^k(s))+\lambda\sum_{s'}p^k_{s^k(s)s'}V(s') \qquad (4)$$

where λ is the discount factor and $s^k(s)$ is the resulting state from performing action k in state s, i.e., $s^k(s)=(0,\phi(s,k))$.

Equation (3) states that the *DN* action incurs a quality cost for leaving the system in state s without maintenance for the decision epoch of length τ, plus the expected cost given that the system state at the beginning of the current decision epoch is s and may evolve to any state s' with a probability $p^m_{ss'}$. Equation (3) states that performing action k incurs a maintenance cost $c(k)$ in addition to the quality cost resulting from beginning the decision epoch in state $s^k(s)$ plus the expected cost given the system state may evolve from $s^k(s)$ to any state s' with probability $p^k_{s^k(s)s'}$.

Because of the sensitivity of the decision process to θ showed by numerical results, we take the non-obyservability of θ by extending the MDP formulation to a partially observed Markov decision process. This is made by replacing the deterministic knowledge upon the DGR θ (i.e., the approximation with the average speed of deterioration) by a probability distribution over all possible states using the same available information, i.e., observations of ρ. More specifically, we assume that the DGR at the beginning of each decision epoch t is normally distributed with a mean equal to the average cracking speed $\bar{\theta}=\rho_{t+\tau}-\rho_t$ and a given standard deviation σ. Nevertheless, POMDPs are known to suffer from the ``curse of dimentionality'', which make the definition of heuristic solution procedure necessary. For sake of simplicity, we do not present the mathematical formulation of the POMDP problem. The reader can refer to [11] and [12] for the detailed model.

Application to the road maintenance problem

We apply the bivariate, state-dependent deterioration model developed in Section 1 to the road pavement problem. We consider a road section subject to fatigue deterioration because of variable traffic loads and random, harsh environment changes. More specifically, we are interested in the longitudinal cracking deterioration mode That is a two-phase process operating as follows [2]. The traffic load applied to the pavement surface generates tensile stress at the bottom of the asphalt layer that leads the road tensile strength to deteriorate overtime. Micro-cracks appear at weak spots at the road bottom when the stress applied by the traffic load exceeds the tensile strength of the road. These micro-cracks then propagate in the inferior layers of the road until reaching the road surface to give way to surface longitudinal cracks that continue propagating on the surface of the road.

The road section is periodically inspected to yield a measure of its longitudinal cracking percentage ρ represented by Figure 1. Not only ρ does not represent the underlying deterioration of inferior layers, but also does not take into account either the number or the distribution of surface cracks. Hence we propose to introduce the deterioration growth rate θ to model the underlying deterioration and to use the deterioration model presented in Section 1.

Maintenance actions, denoted $k=1,\dots,R$, consist in renewing partially or completely a certain thickness, i.e., milling a cracked thickness and resurfacing with a new layer having the same thickness. A perfect action consists in renewing the total road thickness. Maintenance actions conceal surface cracks (i.e., reset ρ to zero) and reduce the DGR according to a deterministic function $\phi(s,k)$ where $s=(\rho,\theta)$ is the road section state.

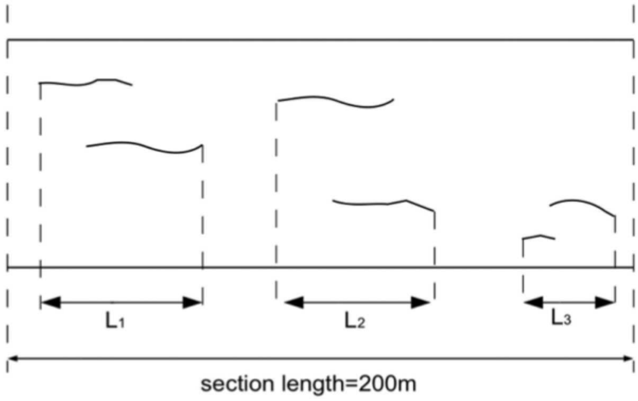

Figure 1 : *Longitudinal cracking percentage metric for 200m road section LCP=(L1+L2+L3)/200.*

A. The deterioration model

In order to to highlight the state-dependent character of the deterioration model, as well as the effect of introducing the DGR as a deterioration parameter, we consider the following shape function γ that has a form inspired by the well-known Gaussian function:

$$\gamma(\tau,\rho,\theta,m)=b_1 r_m \theta^2 \tau \exp(r_m(\rho-(1-\theta))^2)$$

where b_1 is a constant and r_m is the reduction factor of action m .
Figure 2 shows the variations of the expected deterioration in ρ for different levels of θ . Note that at the beginning of the propagation phase the expected increase in deterioration increases as ρ increases, but with decelerated variation. Whereas, when the DGR level becomes higher, the expected increase in cracking level increases and then decreases quickly, as the DGR is influential. This reflects the fact that, for a single road section, the probability of increasing the cracking level of the section is higher at the beginning of the propagation phase, and decreases as the section becomes more cracked.

B. Optimal maintenance policies

We consider, in addition to the *DN* and the perfect action, three different imperfect actions. The maintenance effect function ϕ is given by :

$$\phi(s,k)=\theta_0+r_k(\alpha_1\rho+\alpha_2\theta), k=1,\dots,4.$$

where $(\alpha_1,\alpha_2)\in[0,1]$ verify $\alpha_1+\alpha_2=1$.
The quality cost function is given by

$$C_q(\tau,s)=c_1\tau(\alpha_1\rho+\alpha_2\theta)$$

where c_1 is a constant.

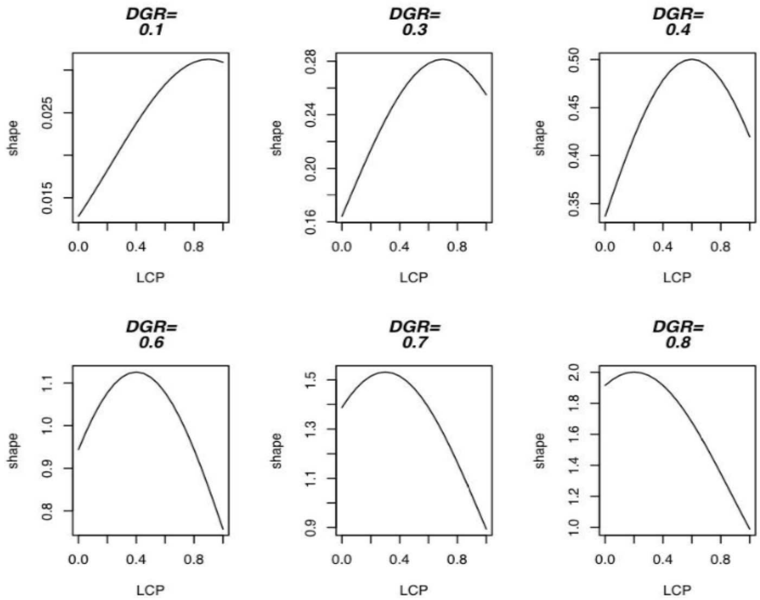

Figure 2: *Variations of the form shape function in ρ for different levels of θ .*

Finally, using the proposed deterioration model to construct the state transition matrix of the state for the MDP model we derive the following decision matrices. The first one (left) is the decision matrix given that the last performed action is of type 2, whereas the second one (right) is for $m=4$.
Both matrices give the optimal maintenance action to perform for each possible road section state. For example, if the observed section state is $(\rho=0.6, \theta=0.1)$ and $m=2$, then the action advised is $k=2$, whereas for the same deterioration level, if $m=4$, then the optimal action is $k=1$.

Numerical examples show the significance of considering the DGR θ as a deterioration and decision parameter. For the same LCP level, optimal policies give different actions for different levels of θ . Moreover, it allows deriving preventive policies that advise maintenance even when the road surface is free of cracks.

m=2

ρ										
0.9	3	3	4	4	4	4	4	4	4	4
0.8	3	3	3	3	3	3	3	3	3	3
0.7	2	2	2	2	2	2	2	2	2	2
0.6	1	2	2	2	2	2	2	2	2	2
0.5	1	1	1	1	1	1	1	1	1	1
0.4	1	1	1	1	1	1	1	1	1	1
0.3	1	1	1	1	1	1	1	1	1	1
0.2	1	1	1	1	1	1	1	1	1	1
0.1	1	1	1	1	1	1	1	1	1	1
0.03	1	1	1	1	1	1	1	1	1	1
0.0	0	0	1	1	1	1	1	1	1	1
θ	**0.03**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**

m=4

ρ										
0.9	3	3	3	3	3	4	4	4	4	4
0.8	2	3	3	3	3	3	3	3	3	3
0.7	2	2	2	2	2	2	2	2	2	2
0.6	1	1	1	2	2	2	2	2	2	2
0.5	1	1	1	1	1	1	1	1	1	1
0.4	1	1	1	1	1	1	1	1	1	1
0.3	1	1	1	1	1	1	1	1	1	1
0.2	1	1	1	1	1	1	1	1	1	1
0.1	1	1	1	1	1	1	1	1	1	1
0.03	0	0	1	1	1	1	1	1	1	1
0.0	0	0	0	0	0	1	1	1	1	1
θ	**0.03**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**

Figure 3 : *Optimal maintenance policies for the road pavement problem.*

Comparison of the two decisions matrices shows the effect of changing the deterioration process after different actions. Note that for low deterioration levels, it is optimal to do nothing after a total renewal, i.e., $m=4$, but the road should be maintained after an imperfect action $m=2$.

Derived policies indicate the existence of a control limit structure with respect to both deterioration model. Note that the policies are monotone in that optimal actions become stronger as the road deterioration level is higher. This property is intuitive and is very useful from a practical point of view as well as a computational point of view since it helps accelerating classical solution procedure of the MDP model and define efficient heuristic one for the POMDP model.

In the case of POMDPs, optimal actions can be given for any belief state in θ , given the observed ρ . For example, if $\rho=0.4$ and θ can take five possible values $(0.03, 0.2, 0.4, 0.6, 0.8)$ with the respective probabilities $(0.05, 0.2, 0.35, 0.3, 0.1)$ then the optimal action to perform is of type 3.

Summary & Conclusion

This paper presents a novel deterioration model for cumulative, two-phase deteriorating systems. More specifically, a new deterioration parameter in addition to the observable one is introduced in order to take into account the underlying deterioration in both initiation and propagation phases. therefore, the proposed model is based on the joint evolution of observable and underlying deterioration. A special feature about this model is to be state-dependent, i.e., non-stationnary in state, since we make the process increments depend on the current deterioration level instead of age. Moreover, whereas all maintenance actions have the same effect on the parameter of observable deterioration, introducing the new deterioration parameter allows modeling this different impacts by reducing the level of the deterioration growth rate as well as changing the deterioration process.

We incorporate this deterioration model into a Markov decision process and derive optimal maintenance policies. Numerical examples confirm the sensitivity of the decision rules to the underlying deterioration θ as well as the interest of relaxing the classical assumption of the same deterioration process.

Finally, in order to take into account the non-observability of new deterioration parameter, we extend the formulation of the optimization problem to a partially observed Markov decision process. However this extension make the resolution of the model more difficult.

References

[1] M. Abdel-Hameed. A gamma wear process. IEEE Transaction on Reliability, 24(2):152–153, 1975.

[2] B. Castanier and T. G. Yeung. Optimal highway maintenance policies under uncertainty. In Proceedings of the Annual Reliability and Maintainability Symposium, 2008.

[3] M. L. Puterman. Markov decision processes: discrete stochastic dynamic programming. Wiley Series in Probability and Mathematical Statistics: Applied Probability and Statistics. John Wiley and Sons Inc., New York, 1994.

[4] S. Sheu. Optimal block replacement policies with multiple choice at failure. Journal of Applied Probability, 29(1):129–141, 1992.

[5] J. M. van Noortwijk. Optimal maintenance decisions for hydraulic structures under isotropic deterioration. PhD thesis, Delft University of Technology, Delft, 1996.

[6] J. M. van Noortwijk and H. E. Klatter. Optimal inspection decisions for the block mats of the eastern-scheldt barrier. Reliability Engineering & System Safety, 65(3):203 – 211, 1999.

[7] J. M. van Noortwijk and Dan M. Frangopol. Two probabilistic life-cycle maintenance models for deteriorating civil infrastructures. Probabilistic Engineering Mechanics, 19(4):345 – 359, 2004.

[8] J. M. van Noortwijk. A survey of the application of gamma processes in maintenance. Reliability Engineering & System Safety, 94(1):2 – 21, 2009. Maintenance Modeling and Application.

[9] J. M. van Noortwijk, J.A.M. van der Weide, M.J. Kallen, and M.D. Pandey. Gamma processes and peaks-over-threshold distributions for time-dependent reliability. Reliability Engineering & System Safety, 92(12):1651 – 1658, 2007. Special Issue on ESREL 2005.

[10] H. Wang. A survey of maintenance policies of deteriorating systems. European Journal of Operational Research, 139(3):469 – 489, 2002.

[11] M. Zouch, T. G. Yeung, and B. Castanier. Optimal resurfacing decisions for road maintenance : A pomdp perspective. In Proceedings of the Annual Reliability and Maintainability Symposium (RAMS), Florida, USA, 2011. RAMS.

[12] M. Zouch, T. G. Yeung, and B. Castanier. Structural properties to solve MDP and pomdp models for road maintenance optimization. Technical report, Ecole des Mines de Nantes, France, 2011.

[13] M. Zouch, T. G. Yeung, and B. Castanier. Two-phase state-dependent deterioration model for maintenance optimization. Submitted to Naval Reasearch Logistics, 2011.